Beyond Coupons by James Grosjean

Visitors to Las Vegas are often given "fun books" with coupons providing discounts on everything from Hoover Dam excursions to hot dogs. Usually a few of the coupons offer improved payoffs in casino games. For example, with a $10 "match-play" coupon, the gambler wagers $10 of his own money (or chips) along with the coupon, and winners are paid as if his bet were $20. By changing the payoffs of the game, these coupons can affect the player's strategy in subtle ways.

For example, suppose you had a $10 match-play coupon good for the game of baccarat. Suppose that the coupon is valid for one hand only, surrendered even in case of a tie. What bet would you make? You might recall from reading Griffin's *The Theory of Blackjack* that the expectations for the 8-deck American baccarat game are -1.058%, -1.235%, -14.360% for the Banker, Player, and Tie bets, respectively. Bet Banker, eh? *Au contraire.*

Match-play coupons are mathematically equivalent to loss rebates. Say we bet Banker. We'd win $20 (less a standard $1 commission). When we lose, we lose $20, and then get "rebated" $10, implemented via the coupon. The net loss is only $10 of our own money. In *Beyond Counting*, we learned that loss rebates are like financial options, and *variance is good* for options.

In the baccarat example, the high variance of the Tie bet makes it the best use of the match-play coupon. With probability 0.0951560, we win $160 (an 8:1 payoff on our "$20" bet), and with probability 0.9048440, we lose $10, for an expectation of $6.18. Perhaps you feel this was a trick question, because you were assuming that the coupon had the common restriction to even-money bets only. Fair enough—so now you're betting Banker, eh? The expectation on the Banker bet is $0.4585974 \cdot \$19 - 0.4462466 \cdot \$10 = \$4.25$. The expectation on the Player bet is $0.4462466 \cdot \$20 - 0.4585974 \cdot \$10 = \$4.34$.

We should bet our $10 with the even-money match-play coupon on Player! How did this happen? Let's disaggregate each expectation into two parts: a standard $20 wager, and a $10 loss rebate. The $20 wager has an expectation of -$0.21 on Banker and -$0.25 on Player. The loss rebate, though, has an expectation of $4.46 on Banker and $4.59 on Player. Ironically, since the Player wager loses more often than the Banker wager does, we reap the benefit of the loss rebate more often. "Fun books," indeed!

The game of roulette also illustrates these principles. Everyone knows that (except for the dreaded 5-number bet) all the bets have the same expectation, right? If we wager the $10 and coupon on the 00 straight-up, our expectation is $8.68. On Black, our expectation is only $4.21. The high variance of the straight-up number has more than doubled the value of our coupon. Looking at it by disaggregation, we have a $20 wager, and a $10 loss rebate. The $20 wager has the same expectation on a number or on a color, but the $10 loss rebate kicks in on the straight-up wager 37/38 of the time, instead of only 20/38 of the time with the color wager.

Bonuses on Naturals

Let's continue warming up by looking at a coupon that pays 2:1 on the player's first blackjack and is then relinquished. This represents an extra half bet over the typical 3:2 payoff. So, a coupon limited to a $10 wager would be worth $5, less the "bleeding cost" of playing a negative blackjack game while waiting for the natural. Since the probability of an untied natural in a 6-deck game is 0.0453230, we will have to play 22.06386 hands (in expectation) to churn the coupon. In our benchmark 6-deck S17 DOA DAS RS4 noRSA game, we bleed at about 4.06 cents per hand, so the coupon's overall value is reduced to $4.10. Another way to look at it is that if we play a hand with a 2:1 payout on naturals, our edge is 1.86%. In expectation, we'll play 22.06386 hands of this game, for an overall expected profit of $4.10 if our wager is limited to $10. If we were able to play with that edge for an hour, we could play anywhere from 40-200 rounds, earning as much as $37.20 in expectation. For such a coupon with a time restriction, the key is to play the fastest table possible. Look for a continuous shuffling machine or a multiple-deck shoe to minimize downtime.

A minor variation is a coupon that pays 2:1 on the first natural, but 3:1 if that natural is suited. Using the logic above, and the fact that we can expect one quarter of our naturals to be suited, the coupon is worth three quarters of our bet, less the bleeding cost. On a \$10 wager, this amounts to \$7.50 − \$0.90 = \$6.60.

The Free Ace

Another common blackjack coupon is a "free Ace" as the player's initial card. As discussed later, this coupon is not quite as valuable as a real Ace, because the coupon is like a 5th Ace in a 53-card deck. The edge is 50.43% in a 1-deck H17 DOA NoDAS NoRSA game. If the Ace can be kept on pushes, the value is 55.08% of a bet.

The Numbers Are In

There is seemingly no end to the variety of coupons, but I will use five common examples: (1) a \$10 Match-Play coupon, Relinquished after one usage—win, lose, *or tie*—denoted by MPR; (2) a \$10 Match-Play coupon, Saved on a tie, but relinquished after a single win or loss, denoted by MPS; (3) a \$100 Funny chip, Relinquished after one usage—win, lose, or tie—denoted FR; (4) a \$100 Funny chip, Saved on a tie, but relinquished after a single win or loss, denoted FS; (5) a \$100 Non-Cashable chip, saved on a win or tie, but relinquished on a loss—denoted NC. This last type of chip is identical in every way to a regular, live chip, except that it cannot be cashed out. You must play it until you lose it.

There are three main factors influencing the value of a coupon. A match-play where we have to risk live money is not worth as much as its funny chip counterpart, other rules being equal. Being allowed to retain the coupon on pushes (or wins) increases the value. The most important factor, though, is whether the coupon is restricted to even-money bets. Because of the effect of variance in a loss rebate program, the coupon is actually more valuable in expectation if we can use it on bets paying odds. All of those horrible longshot bets are suddenly looking good for a coupon player.

Before we review the charts, let's clarify the notation used. For match-plays, we use the coupon in addition to a \$10 live-money wager. For the funny chips, we use only the \$100-denominated chip. A dollar sign precedes the expectations for even-money bets.

Baccarat. Winning Banker wagers are subject to a 5% commission, levied against a match-play coupon as well. So, the match-play coupon with a \$10 live wager would garner \$19 on a winning Banker hand.

Blackjack. Typical 6-deck rules are used: Stand on soft 17 (S17), Double On Any two cards (DOA), Double After Split (DAS), ReSplit up to 4 hands (RS4), No ReSplitting of Aces (NoRSA), and only one card to split Aces. A full 3:2 payoff on naturals is indicated as "BJ1.5×." "BJ1.25×" indicates that the live-money portion of the wager pays 3:2, while the match-play coupon pays only 1:1. So, an untied natural on the \$10 with match-play coupon would net \$25. The "20/20" in brackets indicates that with \$10 live money and a match-play coupon, a full \$20 in additional live money is used to split or double. The "20/10" means that only \$10 in additional live money is used. Where different, the expectations for optimal play (explained below) and basic strategy are both shown in the last column.

Casino War. The 6-deck game described in *Beyond Counting* is used. For match-play coupons, "20/20" means that an additional \$20 live is required to go to war, accompanying the original \$10 and match-play coupon. For coupons that require only \$10 in additional war expenditures, "20/10" is indicated. The phrase "with Bonus" means that the player wins an extra bet if he ties in the war following an initial tie, for a net win of two bets. If there is "no Bonus," then the player wins only his original bet if he ties in the war. Because the bonus could be interpreted as an even payoff on the additional war expenditures, as opposed to an odds payoff on the initial wager, Casino War probably satisfies the restriction to even-money bets only, whether or not there is a bonus. In the case of surrender, assume that the player keeps his $10, but relinquishes the match-play coupon. The reverse case, where he forfeits his $10 live, but keeps the coupon, is worth only $4.20. If he has to give up only $5, but keep the coupon, the coupon value is $4.60. If the player surrenders a funny chip or non-cashable chip, he *receives* $50 live, and relinquishes the chip. If the surrender implementation requires him to *pay* $50 live, and keep possession of the chip, then fighting is a better option.

Craps. For odds wagers, a typical 3/4/5 limit is used. So, in the table for the non-cashable chip (NC), a $17 live wager is put on the Don't Pass, and an odds wager of NC+$2 is made if a point is established. If the shooter sevens out, the odds wager will pay $51, $68, or $85, depending on whether the point is 4-10, 5-9, or 6-8, respectively. For Field wagers, the payoffs for the 2 and the 12 are shown in parentheses.

Let It Ride. The coupon is placed on the blind bet, the one that cannot be pulled back. The "20/10/10" notation means that $10 live is placed on the blind bet along with the $10-denominated match-play coupon, so that winning hands will be paid on a $20 base. For the first two optional bets, only $10 live can be used. The "20/20/20" notation refers to cases where $20 live can be used to back up the coupon amount as well. Three common payout schedules are shown.

Roulette. Expectations are shown for single-zero roulette (preceding the slash), and double-zero roulette (following the slash). Color wagers (Red or Black) are shown as "18-number" wagers. Straight-up wagers are listed as "1-number" wagers. There is no *en prison* rule in effect.

Three Card Poker. Ante bonuses for a Straight, Three of a Kind, and Straight Flush are shown with slashes in the "Wager" column of the table. The full payout is 1/4/5. Stingy casinos may use 1/3/4. The notation "Play 1/3/4" means that the funny chip is used on the Play wager, but that the live money on the Ante earns bonuses of 1/3/4. For chips that are restricted to even-money payouts only, the "0/0/0" indicates that there are no Ante bonuses. A "0.5/2/2.5" indicates a full payout on the live money wager, but no Ante bonuses on the match-play coupon. Under those conditions a Straight would pay an Ante bonus of $10, and, assuming the hand wins, another $20 for the Ante itself. The playing strategy is shown in parentheses; we Play any hand at least as good as the one indicated. For example, "(Q76)" would instruct us to make a Play wager with any hand Q76 or better, but fold otherwise. We never fold if the strategy is shown as "(532)," the weakest hand in the game. The "$$" notation means that live money should be used, so that the coupon can be saved for a subsequent hand. The notation "20/20" in brackets means that the $10 with the match-play on the Ante is backed by a full $20 live on the Play, or vice versa. "20/10" means that the $10 with the match-play on the Ante is backed by only $10 live money on the Play. Finally, "10/20" means that $10 live money is placed on the Ante, and then $10 live with the match-play is placed on the Play.

Match-Play Mania

As the chart shows, the best usage of a match-play coupon is straight-up on a number in roulette, if wagers paying odds are permitted. The value is $8.68 on a double-zero wheel, still roughly double the value of using the coupon in blackjack. If odds bets (including the 3:2 payout on blackjacks) are disallowed, then the craps Pass line becomes a contender at $4.788 (perhaps a surprise to "savvy" craps players who bet the Don't). If the match-play is saved on ties, which seems to be more prevalent than the alternative, then Three Card Poker becomes an attractive possibility. The strategy is to place $20 live on the Ante. On any hand A76 or weaker, place an additional $10 live with the match-play coupon on the Play bet; on any hand A82 or stronger, use a full $20 live cash on the Play, saving the match-play coupon in your pocket for use on a subsequent hand. This approach earns an expected $5.36, and should satisfy the even-money restriction, since the Play bet never pays more than 1:1.

Notice that the typical blackjack coupon (no Surrender, even-money blackjacks, 20/10 on splits and doubles) is worth only $4.71. Even if the coupon cannot be used on the Play bet of Three Card

Poker, the baccarat Player bet would still produce a better return at $4.80 (perhaps a surprise to "savvy" baccarat players who bet Banker).

The thing to remember is that blackjack is rarely the best game to play when moving a match-play coupon. (You're probably best off minimizing your exposure in the blackjack pit anyway.) For years blackjack counters have scoffed at roulette, the Big Six wheel, and other carnival games, but these games have a purpose after all.

Match-Play Values in Various Games

Game	Wager	Expectation
MPR: $10-denominated Match-Play coupon, Relinquished on tie		
0/00 Roulette	1-number (straight up)	9.19/8.68
0/00 Roulette	2-number	8.92/8.42
0/00 Roulette	3-number	8.65/8.16
0/00 Roulette	4-number	8.38/7.89
0/00 Roulette	6-number	7.84/7.37
0/00 Roulette	5-number	7.57/7.11
Craps	3 (or 11) 15:1	7.22
Let It Ride	Blind 1/2/3/5/9/15/40/100/200 [20/20/20]	7.01
Three Card Poker	Pair Plus 1/4/6/30/40	6.98
Craps	2 (or 12) 30:1	6.94
Let It Ride	Blind 1/2/3/5/10/15/25/100/500 [20/20/20]	6.93
Let It Ride	Blind 1/2/3/5/8/11/50/200/1000 [20/20/20]	6.91
Big Six	Joker 45:1	6.85
Three Card Poker	Pair Plus 1/4/6/25/40	6.74
Craps	Any craps 7:1	6.67
Baccarat	Tie 8:1	6.18
Three Card Poker	Pair Plus 1/3/6/30/40	5.98
0/00 Roulette	12-number (column)	6.22/5.79
Three Card Poker	Ante 1/4/5 (532) [20/10]	5.20
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.5× [20/20]	5.19
Craps	Any 7 4:1	5
Craps	Field (2×, 3×)	5
Big Six	Joker 40:1	5
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.5× [20/10]	4.94
Three Card Poker	Ante 1/4/5 (QT2) [20/20]	4.92
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.25× [20/20]	$4.89
Casino War	6-deck with Bonus (Fight) [20/10]	$4.85*
Craps	Pass	$4.788
Casino War	6-deck no Bonus (Fight) [20/10]	$4.74
Blackjack	S17 DOA DAS RS4 noRSA BJ1.5× (optimal,BS) [20/20]	4.73,4.71
Three Card Poker	Ante 0.5/2/2.5 (532) [20/10]	$4.67
Craps	Don't Pass	$4.66
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.25× [20/10]	$4.64
Casino War	6-deck (Surrender)	$4.63
Blackjack	S17 DOA DAS RS4 noRSA BJ1.5× (optimal,BS) [20/10]	4.54,4.53
Casino War	6-deck with Bonus (Fight) [20/20]	$4.51*
Blackjack	S17 DOA DAS RS4 noRSA BJ1.25× (optimal,BS) [20/20]	$4.50,4.49
Craps	Field (2×, 2×)	4.44
Casino War	6-deck no Bonus (Fight) [20/20]	$4.40
Three Card Poker	Ante 0.5/2/2.5 (QT2) [20/20]	$4.39
Three Card Poker	Play (Fold 532-Q63; $$ on Q64-AKJ; MPR on 223+) [10/20]	$4.36
Baccarat	Player	$4.34
Blackjack	S17 DOA DAS RS4 noRSA BJ1.25× (optimal,BS) [20/10]	$4.31,4.30
Baccarat	Banker 5%	$4.25
0/00 Roulette	18-number (Black)	$4.59/4.21
Three Card Poker	Play (MPR on 532-A73; $$ on A74+) [20/20]	$3.75
Let It Ride	Blind 1/2/3/5/9/15/40/100/200 [20/10/10]	3.61
Let It Ride	Blind 1/2/3/5/10/15/25/100/500 [20/10/10]	3.56
Let It Ride	Blind 1/2/3/5/8/11/50/200/1000 [20/10/10]	3.53
Craps	Field (1×, 1×)	$3.33
MPS: $10-denominated Match-Play coupon, Saved on tie		
Three Card Poker	Play (Fold 532-Q63; $$ on Q64-AKJ; MPS on 223+) [10/20]	$6.26
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.5× [20/20]	5.69
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.5× [20/10]	5.41
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.25× [20/20]	$5.36
Three Card Poker	Play (MPS on 532-A76; $$ on A82+) [20/20]	$5.36
Three Card Poker	Ante 1/4/5 (532) [20/10]	5.20
Blackjack	S17 DOA DAS RS4 noRSA BJ1.5× [20/20]	5.16
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.25× [20/10]	$5.09
Blackjack	S17 DOA DAS RS4 noRSA BJ1.5× [20/10]	4.96
Three Card Poker	Ante 1/4/5 (QT2) [20/20]	4.92
Blackjack	S17 DOA DAS RS4 noRSA BJ1.25× [20/20]	$4.91
Baccarat	Player	$4.80
Craps	Don't Pass	$4.79
Blackjack	S17 DOA DAS RS4 noRSA BJ1.25× [20/10]	$4.71
Baccarat	Banker 5%	$4.70
Three Card Poker	Ante 0.5/2/2.5 (532) [20/10]	$4.67
Three Card Poker	Ante 0.5/2/2.5 (QT2) [20/20]	$4.39

Expectations for double-zero roulette are shown following the slash.
In games with no ties, examples are shown only in MPR section.
In 6-deck blackjack, probability of natural is 0.0474895.
Probability of natural is 0.0453230, accounting for 22.6615-cent gap
for coupons relinquished after a single usage.

Funny-Chip Follies

We see the same phenomena when using "funny chips," by which I mean chips that are relinquished when a decision is generated, win or loss. Roulette is still the juggernaut, followed by various carnival games and bets. If payouts are restricted to even-money bets, then Three Card Poker emerges victorious, whether or not the chip is saved on a tie. For chips relinquished on ties, the strategy is to use \$100 live on the Ante. Then, on any hand K87 or weaker, use the chip on the Play bet; on any hand K92 or stronger, we use cash on the Play bet, and keep the chip in our pocket for a subsequent hand. This produces an expectation of \$51.43, a shock to those who thought that \$50 was an upper bound on the value of this type of chip. If the chip is saved on ties, then Three Card Poker produces a whopping \$73.15, compared to the sub-\$50 values of using the chip in blackjack with even-money payoffs.

Funny-Chip Values on Various Games		
Game	Wager	Expectation
FR: $100-denominated Funny chip, Relinquished on tie		
0/00 Roulette	1-number (straight up)	94.59/92.11
0/00 Roulette	2-number	91.89/89.47
0/00 Roulette	3-number	89.19/86.84
0/00 Roulette	4-number	86.49/84.21
Craps	3 (or 11) 15:1	83.33
Craps	2 (or 12) 30:1	83.33
Big Six	Joker 45:1	83.33
0/00 Roulette	6-number	81.08/78.95
0/00 Roulette	5-number	81.08/78.95
Craps	Any craps 7:1	77.78
Baccarat	Tie 8:1	76.12
Big Six	Joker 40:1	74.07
Let It Ride	Blind 1/2/3/5/9/15/40/100/200	73.09
Let It Ride	Blind 1/2/3/5/10/15/25/100/500	72.70
Let It Ride	Blind 1/2/3/5/8/11/50/200/1000	72.62
Three Card Poker	Pair Plus 1/4/6/30/40	72.07
Three Card Poker	Pair Plus 1/4/6/25/40	70.90
Three Card Poker	Pair Plus 1/3/6/30/40	67.11
Craps	Any 7 4:1	66.67
0/00 Roulette	12-number (column)	64.86/63.16
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.5×	57.98
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.0×	$55.72
Three Card Poker	Ante 1/4/5 (K32)	53.66
Craps	Field (2×, 3×)	52.78
Three Card Poker	Play (FR on 532-K87; $$ on K92+)	$51.43
Three Card Poker	Play 1/3/4 (FR on 532-K94; $$ on K95+)	$50.50
Casino War	6-deck (Surrender)	$50
Craps	Field (2×, 2×)	50
Craps	Pass	$49.29
Three Card Poker	Ante 0/0/0 (K32)	$48.37
Blackjack	S17 DOA DAS RS4 noRSA BJ1.5× (optimal,BS)	48.05,47.55
Craps	Don't Pass	$47.93
Casino War	6-deck with Bonus (Fight)	47.40
0/00 Roulette	18-number (Black)	$48.65/47.37
Casino War	6-deck no Bonus (Fight)	$46.85
Blackjack	S17 DOA DAS RS4 noRSA BJ1.0× (optimal,BS)	$45.79,45.28
Baccarat	Player	$44.62
Craps	Field (1×, 1×)	$44.44
Baccarat	Banker 5%	$43.57
FS: $100-denominated Funny chip, Saved on tie		
Three Card Poker	Play (FS on 532-K96; $$ on K97+)	$73.15
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.5×	60.46
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.0×	$58.08
Three Card Poker	Play (532)	$54.58
Three Card Poker	Ante 1/4/5 (K32)	53.68
Blackjack	S17 DOA DAS RS4 noRSA BJ1.5× (optimal,BS)	52.26,52.03
Blackjack	S17 DOA DAS RS4 noRSA BJ1.0× (optimal,BS)	$49.78,49.55
Baccarat	Player	$49.32
Craps	Don't Pass	$49.30
Three Card Poker	Ante 0/0/0 (K32)	$48.40
Baccarat	Banker 5%	$48.15
Expectations for double-zero roulette are shown following the slash. In games with no ties, examples are shown only in FR section.		

Non-Cashable Chips

The last type of coupon to examine is what I call a "non-cashable chip" that cannot be cashed out, but is played repeatedly until lost. Since this chip is virtually identical to live cash, we see that the rankings are restored to our conventional notion of what the "good" casino games are. With a non-cashable chip, our goal is to churn the chip for the lowest "transaction fee" possible. In expectation, we have to pay some juice to convert the chip to cash. How much juice depends on two things: the edge on the game we play, and the expected number of hands we'll have to play before producing a loss, at which point the chip has finally been converted. Here, blackjack finally emerges as a good game selection. However, with poor blackjack rules, craps becomes superior. By placing $17 live money on the Don't, a $100 non-cashable chip can be put on the odds (along with $2 live to protect against the casino's chiseling by rounding down). Since you're laying odds, the payoffs should satisfy an even-money restriction.

Some people wrongly reason that although the chip cannot be cashed, it behaves just like a live chip once the player has committed to playing a game by laying that chip on the felt. The flaw in this reasoning is the omission of the fact that the non-cashing constraint extends beyond the current hand to the future use of the chip. Let's look at roulette as an example. The player might think, "If I'm forced to play, and I decide to play roulette, then it doesn't matter which bet I make, since they all have expectation -5.26%." The chart below shows that betting the 00 straight-up is 15 cents superior to betting the 0/00 two-number bet. We already mentioned that the cost of churning the chip depends on the edge, *and* the expected number of hands to lose the chip. For every spin, we're "paying" $5.26 in juice, but by betting the 00 straight-up, we get this thing over quickly. Unless you're Mr. Lucky, you're done after one spin! When betting a color, the value is only $90. We're subjected to the same $5.26 fee per spin, but by betting a color, it takes us more spins to lose.

Another difference between the non-cashable chip and live money emerges in games where initial bets must be matched with additional cash. For instance, suppose we place the non-cashable chip on the Ante in Three Card Poker. Basic strategy is to play Q64 or better. Now suppose we are dealt a Q64. Should we Play? We know that the non-cashable chip must be worth less than its $100 denomination. It turns out that it's as if we have $93.88 in cash on the Ante (that's the true value of the chip). To stay in, we must back up that Ante with a full $100 live cash on the Play. In poker terms, it has become more expensive to call. We need a better hand to justify calling. We should fold the Q64 and Q65. With Q72, we would Play. Now suppose that we are dealt Q72, and the boss, not seeing our hand yet, informs us that Ante Bonuses will not be paid on the non-cashable chip. The result is that we now should fold the Q72! We do not have a Straight or better, so how does the removal of Ante Bonuses affect our decision? If we win or push the hand, we retain the chip for the subsequent hand. Perhaps we'll get a Straight or better on a future hand. So, the removal of Ante Bonuses lowers the value of a non-cashable chip to $84.34, again making calling more expensive. Since we must back up the chip with a full $100 live cash, we need Q76 or better to Play.

This matching principle could affect strategy on splits and doubles in blackjack, but I did not discover any such changes.

In practice, there are at least three ways to churn a non-cashable chip without playing, but that is beyond the scope of this article. There are also ways for skilled players to extract the full $100. Let me know if you discover any methods.

Non-Cashable Chip Values in Various Games		
Game	Wager	Expectation
NC: $100-denominated Non-Cashable chip, saved until lost		
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.5×	99.32
Blackjack	S17 DOA DAS RS4 noRSA BJ1.5×	99.15
Craps	$17 on Don't, NC+$2 Odds	$99.14
Craps	$34 on Pass, NC Odds	98.79
Baccarat	Banker 5%	$97.63
Baccarat	Player	$97.31
Craps	Don't Pass	$97.23
Craps	Pass	$97.21
Three Card Poker	Pair Plus 1/4/6/30/40	96.89
Let It Ride	Blind 1/2/3/5/9/15/40/100/200	96.02
Let It Ride	Blind 1/2/3/5/10/15/25/100/500	95.50
Let It Ride	Blind 1/2/3/5/8/11/50/200/1000	95.39
Casino War	6-deck with Bonus (Fight)	$95.31
Three Card Poker	Pair Plus 1/4/6/25/40	95.30
Craps	Field (2×, 3×)	95
Blackjack	LS S17 DOA DAS RS4 noRSA BJ1.0×	94.71
0/00 Roulette	1-number (straight up)	97.22/94.59
0/00 Roulette	2-number	97.14/94.44
0/00 Roulette	3-number	97.06/94.29
Casino War	6-deck no Bonus (Fight)	$94.21
0/00 Roulette	4-number	96.97/94.12
Three Card Poker	Ante 1/4/5 (Q72)	93.88
0/00 Roulette	6-number	96.77/93.75
Casino War	6-deck (Surrender)	$93.11
0/00 Roulette	12-number (column)	96 /92.31
0/00 Roulette	5-number	93.75/90.91
Three Card Poker	Pair Plus 1/3/6/30/40	90.22
0/00 Roulette	18-number (Black)	$94.74/90
Craps	Field (2×, 2×)	90
Craps	3 (or 11) 15:1	88.24
Craps	Any craps 7:1	87.5
Craps	2 (or 12) 30:1	85.71
Three Card Poker	Play (Q32)	$85.58
Big Six	Joker 45:1	84.91
Three Card Poker	Ante 0/0/0 (Q76)	$84.34
Baccarat	Tie 8:1	84.13
Craps	Field (1×, 1×)	$80
Craps	Any 7 4:1	80
Big Six	Joker 40:1	75.47
Expectations for double-zero roulette are shown following the slash.		

Strategy Deviations for Blackjack

The charts show that except for the non-cashable chip, blackjack should never be the game of choice for the coupon player. However, you might find yourself with a coupon that must be played on blackjack, in which case there are some strategy modifications that you could learn.

Because the coupon is not worth its denominated value, the payoff structure of the game is completely changed. The loss rebate aspect reduces the pain of losing the hand, and splits and doubles with additional live money must be reconsidered. As a general rule, the more similar the coupon is to a live chip, the closer the coupon strategy will resemble typical basic strategy. For instance, it turns out that with the non-cashable chip, there are no strategy changes in blackjack (for the set of rules considered here). Coupons that are saved on ties involve fewer strategy changes than their single-usage counterparts. The live-money that accompanies a Match-Play coupon mitigates the strategy deviations as well. Before looking at the strategy deviations, I refer you to the 6-deck basic strategy chart in Appendix B. The chart includes S17/H17 and DAS/NoDAS. Uppercase letters indicate a player advantage. Also included are the critical fractions when doubling for less (if you happen to be out of money). Looking at A5 v. 4 up as an example, you must double for at least 343/1000 of your initial bet for doubling to be the expectation-maximizing play. If you don't have that much money, you should just hit. For the critical fractions, I provide the higher of the S17 or H17 value, and I round all fractions up to the nearest 1000th. Using basic strategy, a player has an edge of -0.41%. Using the deviations below will gain only a penny or so on a $10 match-play coupon. On the funny chips, the gain is slightly more, but perhaps still not worth the effort. In the chart that follows, the notation PC instructs us to sPlit (and resplit) the hand containing the Coupon. A hand with only live money would be split according to basic strategy. For doubles after a split, only the hand containing the coupon would use the doubling deviations shown in the following table. Remember that only *deviations* are shown. So, in the first section of the chart, we see that we should hit 33 v. 2 up in a DAS game. Now we would also hit that hand in a NoDAS game, but because doing so is basic strategy, there is no entry in this deviations chart.

Match-Play Strategy Deviations for Blackjack
6-deck S17/H17

		Pair Splits	
Hard Hands	Soft Hands	Double After Split	No Double After Split
MPR [20/20]: Relinquished after one round = $4.73 ($4.50)			
9 v. 2 D	A7 v. 2 DS	33 v. 2 H	
11 v. A D	A8 v. 4-6 D		33 v. 4 H/P
12 v. 3 H/S		44 v. 4 H/PC	
16 v. T S		88 v. T S	88 v. T S
		88 v. A P/H	88 v. A P/H
		99 v. 7 PC	99 v. 7 PC
MPR [20/10]: Relinquished after one round = $4.54 ($4.31)			
12 v. 3 H/S	A2 v. 5 H	22 v. 8 PC	
16 v. T S	A2 v. 6 H/D	33 v. 8 PC	
	A4 v. 4 H	44 v. 5 H/P	
	A8 v. 6 S		66 v. 2 PC
		66 v. 7 PC	
		77 v. 8 PC	77 v. 8 PC
		99 v. A PC	99 v. A PC
MPS [20/20]: Saved on tie = $5.16 ($4.91)			
9 v. 2 D	A7 v. 2 DS	33 v. 2 H	
	A8 v. 5-6 D		66 v. 3 H/P
		88 v. T H	88 v. T H
		88 v. A P/H	88 v. A P/H
		99 v. 7 PC	99 v. 7 P2
			(99 v. 7 PC3)
MPS [20/10]: Saved on tie = $4.96 ($4.71)			
11 v. A H	A2 v. 5 H		22 v. 3 PC
	A2 v. 6 H/D	22 v. 8 PC	
	A3 v. 5 H/D	33 v. 8 PC	33 v. 8 PC
	A4 v. 4 H	44 v. 5 H	
	A7 v. 2 S	44 v. 6 H/P	
	A7 v. 3 S/DS		66 v. 2 PC
	A8 v. 6 S	66 v. 7 PC	
		77 v. 8 PC	77 v. 8 PC
		99 v. A S/PC	99 v. A S/PC

Coupon value assumes $10 live, along with match-play coupon, and BJ1.5×. Value in parentheses is for BJ1.25×.

PC indicates that the hand with the coupon should be resplit repeatedly, but a hand with only live money should not be split.

Option preceding slash is for S17 game.

Where two options are shown with no slash, the first is the best option (for both S17 and H17 games), and the second is the next-best play, for cases where the first option is unavailable.

Funny-Chip Strategy Deviations for Blackjack
6-deck S17/H17

Hard Hands	Soft Hands	Pair Splits: Double After Split	Pair Splits: No Double After Split
FR: Relinquished after one round = $48.05 ($45.79)			
8 v. 6 D	A2 v. 4 D		22 v. 4 H
9 v. 2 D	A3 v. 4 D	33 v. 2 H	
9 v. 7 D	A7 v. 2 D		33 v. 3-4 H
11 v. A D	A8 v. 2-6 D		33 v. 7 H
12 v. 2-3 S	A9 v. 3-6 D	44 v. 4 PC	
15 v. T S	TA v. 5-6 D		44 v. 6 D
16 v. 9-T S			
16 v. A H/S		66 v. 2 S	66 v. 2-4 S
17 v. A H/S			77 v. 2-3 S
		77 v. T S	77 v. T S
		88 v. 9-T S	88 v. 9-T S
		88 v. A H/S	88 v. A H/S
		99 v. 7 PC	99 v. 7 PC
			99 v. 9 S
		TT v. 3 P2	TT v. 3 P2
		TT v. 4-6 PC	TT v. 4-6 PC
FS: Saved on tie = $52.26 ($49.78)			
8 v. 6 D	A2 v. 4 H/D		22 v. 4 H
9 v. 2 D	A3 v. 4 D	33 v. 2 H	
11 v. A D	A7 v. 2 D		33 v. 4 H
	A8 v. 3-6 D		33 v. 7 H
	A9 v. 4-6 D	44 v. 4 PC	
	TA v. 5-6 D		44 v. 6 D
		66 v. 2 H	
			66 v. 3 H
			77 v. 2 S
		88 v. 9 H	88 v. 9 H
		88 v. T S	88 v. T S
		88 v. A H	88 v. A H
		99 v. 7 PC	99 v. 7 PC
		TT v. 4-6 PC	TT v. 4-6 PC

Coupon value assumes $100-denominated funny chip, and BJ1.5×. Value in parentheses is for BJ1.0×.

PC indicates that the hand with the coupon should be resplit repeatedly, but a hand with only live money should not be split. P2 indicates splitting to two hands only.

Option preceding slash is for S17 game.

Where two options are shown with no slash, the first is the best option (for both S17 and H17 games), and the second is the next-best play, for cases where the first option is unavailable.

TA indicates a Ten and an Ace obtained after splitting Tens.

Surrender

Some casinos offer Late Surrender, a rule allowing a player to forfeit half his wager on his initial two-card hand, after the dealer has verified that she does not have blackjack. Even the fine print does not explain how this rule is applied to coupons, so the opinion of the pit boss will carry the day. On a match-play, one interpretation would be to forfeit half of the live money, along with the entire coupon. With this implementation, the player should never surrender, because he is giving up too much.

If, however, the player forfeits half of his live money, but saves the coupon for another hand, the rule adds significantly to the value of the coupon. In the case of funny chips and non-cashable chips, we can also consider a surrender implementation where the player forfeits the chip, but *receives* half of its denominated value in live chips. Surrender then produces a lower bound of $50 for the value of a $100-denominated chip. The player could take the lock bet of $50 by surrendering any hand (except naturals), but his expectation is higher if he elects to play out any hand with expectation greater than $50.

With match-plays and funny chips, it now becomes optimal to surrender most stiffs against strong upcards. The surrender deviations from basic strategy for 6-deck S17 and H17 games are shown in the following charts. The value of the coupon is then provided for the benchmark 6-deck LS S17 DOA DAS RS4 NoRSA game. In parentheses, the value is shown for coupons paying only even money on blackjacks.

An interesting phenomenon shown on the table is that the playing strategy itself changes depending on the blackjack payoff. Take a common Match-Play coupon Saved on ties, where doubles and splits can back only the live money. The MPS 20/10 section of the table shows that the decision to surrender a hard 13 against an 8 up hinges on whether blackjacks pay 3:2 on the coupon! What does the blackjack payoff have to do with the 13 v. 8 decision? If we surrender, we save the coupon for future hands. A 3:2 payoff on blackjack makes the coupon more valuable for those future hands, so saving the coupon by surrendering now becomes a more enticing option, better than hitting 13.

Even a non-cashable chip changes the payoff structure of the game, so that there could be deviations from basic strategy in principle. As it turns out, the only deviation I have found is that 88 v. T up should be surrendered in a 6-deck NoDAS game where blackjacks pay even money. There are noticeable strategy changes for Three Card Poker, but that would be a poor game choice for a non-cashable chip.

There are a few other surrender implementations that are easy to analyze. Suppose that surrender gives you a choice of keeping your $10 live and giving up the coupon, or vice versa. Since the $10 cash is way more valuable than the coupon, you would elect to relinquish the coupon. If you had no choice and were forced to give up the $10 cash and keep the coupon, then you would never surrender.

With funny chips, a subtle variation is for the player to give the casino $50 live, but keep possession of the chip. If we compare this approach to the one above, where the player *receives* $50 live but gives up the chip, the player effectively pays $100 live in order to retain possession of the $100-denominated chip. That's a bad choice for the player, because the chip is worth less than $100. Surrendering would no longer be optimal with funny chips. With non-cashable chips, the basic strategy surrenders would still be correct, except for 16 v. 9 up, and 15 v. T up. These hands would be hit. The value of the non-cashable chip drops from $99.32 to $99.26. The horror—the horror.

Summary of Surrender Implementations		
Cash Transfer	Disposition of Coupon	Comment
	$10 Match-Play Coupons	
Lose $5	Reduced to $5	Non-existent rule
Lose $5	Relinquished	Do not surrender
Lose $5	Saved	See chart LSI
No transfer	Relinquished	See chart LSII
Lose $10	Saved	Do not surrender
	$100 Funny Chips	
Lose $50	Saved	Do not surrender
Receive $50	Relinquished	See chart LSIII
	$100 Non-Cashable Chips	
Lose $50	Saved	BS, except hit 16 v. 9 and 15 v. T
Receive $50	Relinquished	BS, except surrender 88 v. T in 6-deck NoDAS BJ1.0×

LSI. Match-Play Surrender Deviations for Blackjack 6-deck S17/H17		
Non-Pair Hard Hands		
Little	Big	Pairs
MPR 20/20: Relinquished = [$5.19] ($4.89)		
[5 v. T R]	12 v. 9-A R	[22 v. A H/R]
5 v. A H/R	13 v. 8-A R	[33 v. 9 R]
[6 v. 9 R]	14 v. 7-A R	33 v. T R
6 v. T R	15 v. 7-A R	[33 v. A R]
[6 v. A R]	16 v. 7-A R	(33 v. A H/R)
(6 v. A H/R)	17 v. 8-A R	66 v. 9-A R
[7 v. 9 R]		77 v. 8-A R
7 v. T-A R		88 v. 9-A R
MPR 20/10: Relinquished = [$4.94] ($4.64)		
[5 v. T R]	12 v. 9-A R	33 v. T R
[5 v. A H/R]	[13 v. 8 R]	[33 v. A R]
6 v. T R	13 v. 9-A R	(33 v. A H/R)
[6 v. A R]	[14 v. 7 R]	66 v. 9-A R
(6 v. A H/R)	14 v. 8-A R	77 v. 8-A R
[7 v. T R]	15 v. 7-A R	88 v. 9-T R
[7 v. A R]	16 v. 7-A R	[88 v. A R]
(7 v. A H/R)	17 v. 8-A R	(88 v. A P/R)
MPS 20/20: Saved on tie = [$5.69] ($5.36)		
5 v. T R	12 v. 9-A R	[22 v. A H/R]
5 v. A H/R	[13-16 v. 2 R/S]	[33 v. 9 R]
[6 v. 9 R]	13 v. 8-A R	33 v. T R
6 v. T R	14 v. 7-A R	[33 v. A R]
[6 v. A R]	15 v. 7-A R	(33 v. A H/R)
(6 v. A H/R)	16 v. 7-A R	66 v. 9-A R
7 v. T-A R	17 v. 8-A R	77 v. 8-A R
		88 v. 9-A R
		[*99 v. 7 S]
MPS 20/10: Saved on tie = [$5.41] ($5.09)		
[5 v. T R]	12 v. 9-A R	33 v. T R
[5 v. A H/R]	[13 v. 8 R]	[33 v. A R]
6 v. T R	13 v. 9-A R	(33 v. A H/R)
[6 v. A R]	[14 v. 7 R]	66 v. 9-A R
(6 v. A H/R)	14 v. 8-A R	77 v. 8-A R
[7 v. T R]	15 v. 7-A R	[88 v. 9 R]
[7 v. A R]	16 v. 7-A R	(*88 v. 9 R)
(7 v. A H/R)	17 v. 8-A R	88 v. T R
		88 v. A P/R
Coupon value assumes $10 live, along with match-play coupon, and BJ1.5× in brackets. Values and strategies in parentheses are BJ1.25×. Option preceding slash is for S17 game. On Late Surrender, player loses $5 cash, but saves coupon. * Applies to NoDAS game only.		

LSII. Match-Play Surrender Deviations for Blackjack 6-deck S17/H17		
Non-Pair Hard Hands		Pairs
Little	Big	
MPR 20/20: Relinquished = [$5.14] ($4.92)		
5 v. T R 5 v. A H/R 6 v. T-A R 7 v. T-A R	12 v. 9-A R 13 v. 8-A R 14 v. 7-A R 15 v. 7-A R 16 v. 7-A R 17 v. 8-A R	33 v. T-A R 66 v. 9-A R 77 v. 8-A R 88 v. 9-A R
MPR 20/10: Relinquished = [$4.95] ($4.72)		
5 v. T R 5 v. A H/R 6 v. T-A R 7 v. T-A R	12 v. 9-A R 13 v. 8-A R 14 v. 7-A R 15 v. 7-A R 16 v. 7-A R 17 v. 8-A R	33 v. T-A R 66 v. 9-A R 77 v. 8-A R 88 v. 9-A R
MPS 20/20: Saved on tie = [$5.52] ($5.27)		
6 v. T R 6 v. A H/R 7 v. A H/R	12 v. 9-A R 13 v. 9-A R 14 v. 8-A R 15 v. 7-A R 16 v. 7-A R 17 v. 8-A R	33 v. T R 33 v. A H/R 66 v. 9-A R 77 v. 8-A R 88 v. 9-A R [*99 v. 7 S]
MPS 20/10: Saved on tie = [$5.31] ($5.07)		
6 v. T R (6 v. A H/R) (7 v. A H/R)	12 v. 9-A R 13 v. 9-A R 14 v. 8-A R 15 v. 7-A R 16 v. 7-A R 17 v. 8-A R	33 v. T R 33 v. A H/R 66 v. 9-A R *77 v. 8 R 77 v. 9-A R (*88 v. 9 R) 88 v. T R 88 v. A P/R
Coupon value assumes $10 live, along with match-play coupon, and BJ1.5× in brackets. Values and strategies in parentheses are BJ1.25×. Option preceding slash is for S17 game. On Late Surrender, player keeps cash, but loses coupon. * Applies to NoDAS game only.		

LSIII. Funny-Chip Surrender Deviations for Blackjack
6-deck S17/H17

Non-Pair Hard Hands		Soft	
Little	Big	Hands	Pairs
FR: $100 Funny chip = [$57.98] ($55.72) Relinquished after one round			
5-7 v. 2-A R	12-17 v. 2-A R	A2 v. 2 R	22-44 v. 2-3 R
8 v. 2-4 R	18 v. 2 R	A2 v. 8-A R	*22-44 v. 4 R
8 v. 7-A R	18 v. 8-A R	A3-A5 v. 2-3 R	22-44 v. 7-A R
9 v. 8-A R	19 v. 9-T R	A3-A5 v. 7-A R	55 v. T-A R
10 v. T-A R	19 v. A S/R	A6 v. 2 R	66 v. 2-3 R
		A6 v. 7-A R	*66 v. 4-5 R
		A7 v. 8-A R	*66 v. 6 R/P
		A8 v. 9-T R	66 v. 7-A R
		A8 v. A S/R	77 v. 2-3 R
			*77 v. 4-5 R
			77 v. 7-A R
			*88 v. 2 R
			88 v. 8-A R
			99 v. 9-A R
FS: $100 Funny chip = [$60.46] ($58.08) Saved on tie			
5 v. 2-5 R	12-16 v. 2-A R	A2 v. 9-A R	22 v. 2 R
5 v. 7-A R	17 v. 2-5 R	A3 v. 9-A R	*22 v. 3-4 R
6 v. 2-5 R	17 v. 7-A R	A4 v. 8-A R	*22 v. 7 R
6 v. 6 R/H	18 v. 9-A R	A5 v. 2 R	22 v. 8-A R
6 v. 7-A R		A5 v. 8-A R	33 v. 2-3 R
7 v. 2-4 R		A6 v. 8-A R	*33 v. 4 R
7 v. 7-A R		A7 v. 9-A R	33 v. 7-A R
8 v. 2 R			44 v. 2 R
8 v. 8-A R			44 v. 8-A R
9 v. 9-A R			66 v. 2-3 R
			*66 v. 4-5 R
			*66 v. 6 R/P
			66 v. 7-A R
			77 v. 2 R
			*77 v. 3-4 R
			77 v. 7-A R
			*88 v. 8 R
			88 v. 9-A R
			99 v. 9-A R

Funny chip is denominated at $100, with BJ1.5× in brackets.
Value in parentheses is for BJ1.0×.
Option preceding slash is for S17 game.
On Late Surrender, player receives $50 live chips, and relinquishes funny chip.
* Applies to NoDAS game only.
There are some multi-card totals of 12 that should be hit against a 4 up, but these are insignificant.

Even Money vs. Insurance

In a typical blackjack game where naturals are paid 3:2 odds, taking "even money" is equivalent to insuring a natural against the dealer's Ace.[1] Many match-play coupons, however, do not pay 3:2 on naturals. A $10 wager along with a match-play coupon probably pays a total of $25 on a natural (3:2 on your $10, and 1:1 on your $10-valued coupon). In this case, if you are allowed to take an "even-money" payoff of $20, should you do it? Yes!

The chart below shows that while "even money" costs the player about 4% (80 cents on a $20 wager) in a typical 3:2 game, even money is a *gainer* if the coupon pays only 1:1, *and if* "even money" means a sure-thing payoff of $20.

Sadly, you may be rebuffed with, "You can't take even money, but you can buy insurance." No, thanks. The insurance is a bit costly for my taste, but I have a feeling that there are some coupon users out there who may be interested in the variance reduction of the insurance wager.

Note that there still is such a thing as "even money," if we relax the definition to be "a zero-variance payoff," as opposed to "a zero-variance payoff equal to the original bet." Suppose that we have an original bet b, blackjack payoff p (assuming we do not push), and insurance bet i. When we hold a natural against a dealer's Ace, the payoff is $2i$ if the dealer holds a natural, and $p - i$ otherwise. A guaranteed payoff requires $2i = p - i$, or $i = p/3$. That is, the insurance wager should be one third of the possible blackjack payoff. In a typical game where $p = (3/2)b$, this means that the "even-money" insurance wager is $i = (1/2)b$, and the guaranteed payoff is $2i = b$. In our coupon case, with a $10 wager and $10-valued match-play coupon, the possible blackjack payoff is $25, so the insurance wager would be $\$8\frac{1}{3}$, guaranteeing a payoff of $\$16\frac{2}{3}$.

[If you ever happen to play 6:5 "blackjack," a bastardized single-deck game now prevalent in Las Vegas, and you have a $5 wager riding on a natural against an Ace up, tell the dealer, "Just gimme $4." I'll wager any sum that there is not a single dealer (or pit boss) in Las Vegas who has figured out that an insurance wager $i = (2/5)b$ will produce a guaranteed payoff of $(4/5)b$, which would be $4 on a $5 wager.]

Insuring a Natural on a Match-Play Coupon				
	Expectation/Standard Deviation			
	Number of Decks d			
Amount of Insurance Wager	1	2	6	8
Coupon Pays 3:2 ($30 Total)				
No Insurance	20.82/13.83	20.79/13.84	20.78/13.84	20.77/13.84
$5	20.41/6.91	20.40/6.92	20.39/6.92	20.39/6.92
$\$8\frac{1}{3}$	20.14/2.30	20.13/2.31	20.13/2.31	20.13/2.31
$10	20/0	20/0	20/0	20/0
Coupon Pays 1:1 ($25 Total)				
No Insurance	17.35/11.52	17.33/11.53	17.31/11.54	17.31/11.54
$5	16.94/4.61	16.93/4.61	16.93/4.61	16.92/4.61
$\$8\frac{1}{3}$	16.67/0	16.67/0	16.67/0	16.67/0
$10	16.53/2.30	16.53/2.31	16.54/2.31	16.54/2.31
Probability of Dealer BJ	0.3061	0.3069	0.3074	0.3075
Player has a $10 wager plus match-play coupon. Figures are in dollars. Player holds a natural against a dealer's Ace up. Dealer's blackjack probability is $(16d - 1)/(52d - 3)$.				

[1]Tournament players, among others, are aware of a subtle point—in the case of taking "even money" on a natural, the player does not need to possess any additional chips or cash to make this insurance wager. Saying "even money" is all that is required.

Lock Bets

A vagrant recently approached me at a blackjack table and asked if I would play his $10 match-play coupon for him. He apparently was afraid to put any of his own money at risk. Furthermore, he was unaware of the critical point raised in the previous discussion of insurance: if we have a bet with a large, positive expectation, we may be able to make hedge bets to lower the variance, at the price of lowered expectation. In some cases, the downside risk can be eliminated.[2]

A common method of lowering risk is to use two coupons in the "bet-opposite gambit" (BOG). The technique is applied in a game where two opposing wagers are allowed—baccarat, craps, roulette. Suppose one player wagers $10 and a match-play coupon on Black, and his partner wagers $10 and a match-play coupon on Red. If Black or Red comes up, the winner is paid $20, resulting in a $10 net profit for the coalition. There is a slight risk of a Green number coming up. This risk can be hedged by placing $1 on the 0 and another $1 on the 00. The coalition then wins either $8 or $14. There is no risk of loss.

Did our vagrant need a second coupon to deliver a lock bet? No. Even with only a single match-play coupon, a player who wishes to lock in a profit can do so.[3] The following table shows a few examples of betting schemes that eliminate downside risk. They can be tweaked endlessly, depending on the player's preferences. I chose numbers to make the examples transparent. The two key points are: (1) downside risk can be eliminated, and (2) only the bet with the match-play coupon is positive, while all the hedge bets added to the table are negative. Hedge bets lower the overall expectation, but lock bets can be useful.

Locking a Match-Play Coupon or Funny Chip

Game	Wager	Expectation No Hedge	Expectation With Hedge	Game Outcome: Pass / Black / Banker	Game Outcome: Don't / Red / Player	Game Outcome: 12 / Green / Tie
MPS: $10 + Match-Play Coupon						
Baccarat	$10+MPS on Player, $15 Banker	4.80	4.62	4.25	5	0
Craps	$10+MPS on Pass, $15 Don't, $1 on 12	4.78	4.44	4	4	20
0 Roulette	$10+MPS on Black, $15 Red, $1 on 0	4.59	4.16	4	4	10
0/00 Roulette	$10+MPS on Black, $15 Red, $2 on 0/00	4.21	3.32	3	3	9
FS: $100-valued Funny Chip						
Baccarat	FS on Player, $50* Banker	49.32	48.73	47.5	50	0
Craps	FS on Pass, $50 Don't, $2 on 12	49.29	48.33	48	48	60
0 Roulette	FS on Black, $50 Red, $3 on 0	48.65	47.22	47	47	55
0/00 Roulette	FS on Black, $50 Red, $6 on 0/00	47.37	44.42	44	44	52

The 0/00 bet is a two-number bet paying 17:1
A $51 Banker bet raises the guaranteed payoff to $48.45.

If the funny chip is relinquished even on tied baccarat hands, then the player may wish to hedge further, by placing a $5 tie wager. A profit of at least $40 is guaranteed.

Remember that hedging lowers expectation. However, eliminating downside risk may be an important consideration for a coupon player with a small bankroll, or for a vagrant who *needs* a beer now.

Hedging in Roulette

To further illustrate hedging, let's look at roulette. This game is easy to analyze because there are no ties, and every wager is settled with a single spin (assume that the *en prison* rule is not used).

[2]An example of this can be found in *Beyond Counting,* p. 156.

[3]Pay attention, Copa. Your desperate "Match-play-coupons-may-be-wagered-on-Red-only" restriction won't save you now.

Hedging in Roulette					
Wager (*n*-number)	Unhedged Expectation (Coupon on *n*-number)	Amount of Hedge Wagers: Other *n*-number	Amount of Hedge Wagers: Green	Overall Expectation with Hedge	Possible Net Payoffs
MPR: \$10 + Match-Play coupon on 0/00 Roulette. Expectation=$10 \times (34-n)/38$					
18-number	4.21	15	2	3.32	3/3/9
12-number	5.79	16	3	3.95	5/3/9
6-number	7.37	18	6	2.32	4/2/2
4-number	7.89	–	–	–	160/-10
3-number	8.16	–	–	–	220/-10
2-number	8.42	–	–	–	340/-10
1-number	8.68	–	–	–	700/-10
MPR: \$10 + Match-Play coupon on 0 Roulette. Expectation=$10 \times (35-n)/37$					
18-number	4.59	15	1	4.16	4/4/10
12-number	6.22	16	2	5.30	6/4/28
6-number	7.84	18	3	5.32	7/5/5
4-number	8.38	18	5	4.35	11/3/21
3-number	8.65	18	6	3.14	16/2/2
2-number	8.92	–	–	–	340/-10
1-number	9.19	–	–	–	700/-10
FR: \$100 Funny chip on 0/00 Roulette. Expectation=$100 \times (36-b)/38$					
18-number	47.37	50	6	44.42	44/44/52
12-number	63.16	66	11	55.63	57/55/55
6-number	78.95	83	28	55.63	57/55/61
4-number	84.21	88	44	44.84	52/44/44
3-number	86.84	91	61	30.95	38/30/36
2-number	89.47	–	–	–	1700/0
1-number	92.11	–	–	–	3500/0
FR: \$100 Funny chip on 0 Roulette. Expectation=$100 \times (36-b)/37$					
18-number	48.65	50	3	47.22	47/47/55
12-number	64.86	66	6	61.13	62/60/78
6-number	81.08	83	14	69.49	71/69/75
4-number	86.49	88	22	66.86	74/66/66
3-number	89.19	91	31	61.30	68/60/84
2-number	91.89	94	47	47.43	55/47/47
1-number	94.59	–	–	–	3500/0
Payoff odds are $((36/n)-1):1$, or 1:1, 2:1, 5:1, 8:1, 11:1, 17:1, and 35:1, on 18-, 12-, 6-, 4-, 3-, 2-, and 1-number bets, respectively.					

To see how the chart works, let's examine betting \$10 with a match-play coupon on Column 1 of a double-zero roulette wheel, shown as a twelve-number bet in the second line of the chart. A winner pays 2:1. With no other bets on the table, using the coupon would produce an expectation of \$5.79. Now suppose we need to lock in a positive payoff. By placing \$16 on each of the other two column bets, and \$3 on the 0/00 two-number bet (paying 17:1), our overall expectation is reduced to \$3.95, due to the cost of the \$35 in hedge bets. The benefit is that the possible outcomes of the spin are a net profit of \$5 if our coupon wager wins, \$9 if Green comes up, and \$3 in all other cases. We will walk with at least \$3.

As usual, the (unhedged) expectation of the coupon increases as the variance of the roulette wager increases. The best usage of the coupon is to put it straight-up on a single number. However, note this curiosity: when betting the coupon straight-up, no hedge is possible. Even with a funny chip, worth \$92.11 on a typical wheel, we cannot lock in a single dollar in profit. If we could, it would imply that single-zero roulette is a positive game for the player, an obvious contradiction.

In general, I do not recommend hedge bets when playing a single coupon. Live hedge bets lower our overall expectation, and they can even bring heat. The irony is that casinos send out coupons in order to entice players into making other live bets at a disadvantage. When a coupon player makes hedge bets, that's exactly what he's doing! The hedge bets are all gifts to the casino, and yet many casinos these days try to disallow "opposing wagers." Even when multiple coupons are used to bet opposite, the expectation is no different from using all coupons to load up on one side (for example, using two coupons on Red). The casinos are restricting something they don't understand; they make money in spite of their idiocy.